Production Inventory Model with Deteriorating Items and Price

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Abstract— An Economic Production Quantity (EPQ) model is an inventory control model that determines the amount of product to be produced on a single facility so as to meet a deterministic demand over an infinite planning horizon. In all inventory models a general assumption is that products generated have indefinitely long lives. In general, almost all items deteriorate over time. Often the rate of deterioration is low and there is little need to consider the deterioration in the determination of economic lot size. Nevertheless, there are many products in the real world that are subject to a significant rate of deterioration and age-independent on-going deterioration. In this paper, a dynamic inventory model which is considered with deteriorating production in which each of the production, the demand and the deterioration rates, as well as all cost parameters are assumed to be general functions of time is considered. The objective is to minimize the total net inventory cost. The optimal production cycle time and economic production quantity are derived. The relevant model is built, solved. Necessary and sufficient conditions for a unique and global optimal solution are derived. An illustrative example is provided and numerically verified. The validation of result, this model was coded in Microsoft Visual Basic 6.0

Index Terms: EOQ, Deterioration, Cycle time, Optimality, Price Discount, Shortages, Demand and Production.

1 Introduction

:The primary operation strategies and goals of most manufacturing firms are to seek a high satisfaction to customer's demands and to become a low-cost producer. To achieve these goals, the company must be able to effectively utilize resources and minimize costs. The economic order quantity (EOQ) model was the first mathematical model introduced several decades ago to assist corporations in minimizing total inventory costs. It balances inventory holding and setup costs and derives the optimal order quantity. Regardless of its simplicity, the EOQ model is still applied industry-wide today. In the manufacturing sector, when items are produced internally instead of being obtained from an outside supplier, the economic production quantity (EPQ) model is often employed to determine the optimal production lot size that minimizes overall production/inventory costs. It is also known as the finite production model because of its assumption that the production rate must be much larger than the demand rate. The classic EPQ model assumes thatmanufacturing facility functions perfectly during a production run. However, due to process deterioration or other factors, the generation of imperfect quality items is inevitable. Deteriorating items are common in our daily life; however, academia has not reached a consensus on the definition of the deteriorating items. According to the study of Wee H.M.[1], deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation an so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flower the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value; for the second category, the items have short market life cycle. After a period of popularity in the market, the items loss the original economic value due to the changes in consumer preference, product upgrading and other reasons. The inventory problem of deteriorating items was first studied by Whitin [2], he studied fashion items deteriorating at the end of the storage period. Then Ghare and Schrader [3] concluded in their study that the consumption of the model as stated below: s film and so on

$$\frac{dI(t)}{dt} + \theta I(t) = -f(t)$$

In the function, θ stands for the deteriorating rate of the item, I(t) refers to the inventory level at time t and then f(t) is the demand rate at time t. This inventory model laid foundations for the follow-up study. Raafat [4] and Goyal and Giri [5] made comprehensive literature reviews on deteriorating inventory items in 1991 and 2001 respectively. B.C. Giri and A . Chakraborty (6) considered a single-vendor single-buyer supply chain model where the consumption rate at the retailer depends on the on-hand stock and the production process at the manufacturer is not perfectly reliable. As a result, the machine produces some defective items which have significant impact on the coordinate policy. It is observed that the coordinated policy provides lower cost

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than the non-coordinated policy in all circumstances. Hesham K. Alfares all (7) consider a model and a solution algorithm for incorporating quality and maintenance aspects into a production inventory system for deteriorating items. Closed-form solutions include the quality considerations. Chandra K. Jaggi all (8) describes an inventory model for deteriorating items with imperfect quality has been developed. The screening rate is assumed to be larger than the demand rate. This assumption helps one to meet his demand parallel to the screening process out of the items which are of perfect quality. Vinod Kumar Mishra al (9) describes a production inventory model for time dependent deteriorating item with production distribution and gives analytical solution to determine the optimal production time during normal and disrupted production periods. Factors such as demand, deteriorating rate and so on should be taken into consideration in the deteriorating inventory study. Others factors like price discount, allow shortage or not, inflation and the time-value of money are also important in the study of deteriorating items inventory. By making different combinations of these factors stated above, we can get different inventory models. Price discount is an important strategy which the seller always uses to encourage the buyer to purchase in large quantities; many researchers have taken this factor into consideration in deteriorating items inventory modeling. Therefore, this paper deals with a production inventory model for deteriorating items with shortages. This paper is organized as follows. Section 2 is concerned with assumptions and notations, Section 3 presents mathematical model for finding the optimal solutions and numerical example. Finally, the paper summarizes and concludes in section 4.

2. Assumptions and Notations

2.1 Assumptions: The following assumptions are used to formulate the problem.

1) The demand rate is known, constant and continuous.

2) The lead time is known and constant.

3) Items are produced/ purchased and added to the inventory.

4) The item is a single product; it does not interact with any other inventory items.

5) The production rate is always greater than or equal to the sum of the demand rate.

6) The deteriorating items exist in lot size Q.

2.2 Notations: The following notations are used in our analysis.

1. P – Production rate in units per unit time
2. D – Demand rate in units per unit time
3. Q*-Optimal size of production run
4. C_p – Production Cost per unit
5. θ-rate of deteriorative.
6. x – proportion of defective items from regular production (x is between 0 to 0.1)
7. C_h -Holding cost per unit/year
8. C_0 – Setup cost / ordering cost
9. T – Cycle time
10. T_p - The time during which the stock is building up at a constant rate of P-D units per unit time that is Production time.
11. TC - Total cost

3. Mathematical Model

3.1 The Inventory Equations

The methodology adopted in this paper involves a number of steps. First, the differential inventory equations for all the periods are developed. Next, these differential equations are solved to formulate the cost model. The details of this methodology are discussed below. In order to develop the differential equations, we need to define the two stages of the production inventory cycle shown in figure-1, a simplified representation of the production cycle. The two stages are the production period [0, T_p] and the consumption period $[T_p, T]$.

3.2 Production period [0, T_p]

During this stage, the inventory of good items increases due to production but decreases due to demand and deterioration items. Thus, the inventory differential equation is

$$\frac{dI(t)}{dt} + \theta I(t) = P - D \; ; 0 \le t \le T_P \qquad (1)$$

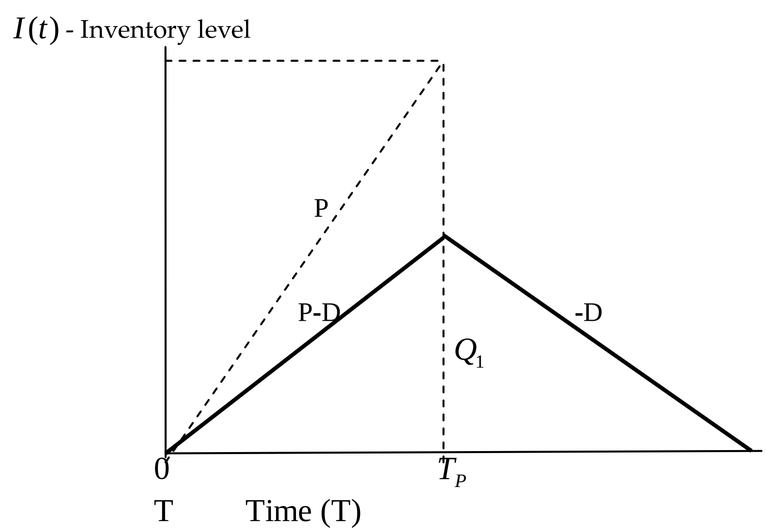

Figure -1 Production Inventory Cycle

3.3 Consumption period $[T_p, T]$

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The inventory differential equation during the consumption period with no production and subsequently reduction in the inventory level due to deterioration items is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -D\,; \quad T_P \le t \le T \qquad (2)$$

with the boundary conditions : I (0) =0 ; I (T) = 0

The solutions of above differential equations are

From (1), $I(t) = \frac{P-D}{\theta}\left[1-e^{-\theta t}\right]; \; 0 \le t \le T_p$ (3)

From (2), $I(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right); \; T_P \le t \le T$ (4)

We know that, $I_1(T_P) = I_2(T_P)$ from the equations (3) and (4) ,

$$\frac{P-D}{\theta}\left[1-e^{-\theta T_p}\right] = \frac{D}{\theta}\left[e^{\theta(T-T_P)} - 1\right]$$

In order to facilitate analysis, we do an asymptotic analysis for $I_i(t)$. Expanding the exponential functions and neglecting second and higher power of θ for small value of θ. Therefore,

$$(P-D)\left[T_p - \frac{1}{2}\theta T_p^2\right] = D\left[(T-T_p) + \frac{1}{2}\theta(T-T_p)^2\right]$$

From **Yong He and Ju He [10]**, T_p was considered as follows,

$$(P-D)\left[T_p - \frac{1}{2}\theta T_p^2\right] = D(T-T_p)\left[1 + \frac{1}{2}\theta(T-T_p)\right]$$

From **Misra [11]** , T_p was considered as follows,

$$T_p = \frac{D}{P-D}(T-T_p)\left[1 + \frac{1}{2}\theta(T-T_p)\right]$$

But in our model, we have considered T_p as follows,

$$(P-D)T_P = D(T-T_P)\text{, Therefore, } T_P = \frac{D}{P}T \qquad (5)$$

Since, $\frac{dT_p}{d\theta} = \frac{1}{2}\cdot\frac{D(T-T_p)^2}{P+\theta D(T-T_p)} > 0$

Assuming $\theta \le 1$, then T_p is increasing in θ. This implies that the manufacturer has to produce more products when deterioration rate increases. Hence, decreasing deterioration rate is an effective way to reduce the product cost of manufacturer.

3.4 Total Cost

The total cost comprise of the sum of the Production cost, Ordering cost, holding cost, Deteriorating cost. They are grouped together after evaluating the above cost individually.

(i) Production Cost /unit time $= P(t)C_P\frac{T_P}{T} = DC_P$ (6)

(ii) Ordering Cost / unit time = $\frac{C_0}{T} = \frac{D}{Q}C_0$ (7)

(iii) Holding Cost / unit time : Holding cost is applicable to both stages of the production cycle, as described by

$$HC = \frac{C_h}{T}\left[\int_0^{T_P} I_1(t)dt + \int_{T_P}^{T} I_2(t)dt\right]$$

$$= \frac{C_h}{T}\left[\int_0^{T_P}\frac{P-D}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_P}^{T}\frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right)dt\right]$$

$$= \frac{C_h}{T}\left[\frac{P-D}{\theta^2}\left(\theta t + e^{-\theta t}\right)_0^{T_P} - \frac{D}{\theta^2}\left(e^{\theta(T-t)} + \theta t\right)_{T_p}^{T}\right]$$

$$= \frac{C_h}{T}\left[\frac{P-D}{\theta^2}\left(\theta T_P + e^{-\theta T_P} - 1\right) - \frac{D}{\theta^2}\left(1 - e^{\theta(T-t)} + \theta(T-T_P)\right)\right]$$

$$= \frac{C_h}{T}\left[\frac{P-D}{\theta^2}\left\{\frac{\theta^2 T_P^2}{2}\right\} + \frac{D}{\theta^2}\left\{\frac{\theta^2(T-T_P)^2}{2}\right\}\right]$$

$$= \frac{C_h}{T}\left[\frac{(P-D)T_P^2}{2} + \frac{D(T-T_P)^2}{2}\right]$$

$$= \frac{C_h}{T}\left[\frac{PT_P^2}{T} + DT - 2DT_P\right]$$

$$= \frac{TC_h D(P-D)}{2P} \text{ from equation (5)} \qquad (8)$$

(iv)Deteriorating Cost/unit time: Deteriorating cost, which is applicable to both stages of the production cycle. Therefore,

$$DC = \frac{C_P}{T}\left[\int_0^{T_p}\theta I_1(t)dt + \int_{T_p}^{T}\theta I_2(t)dt\right]$$

$$= \frac{C_P}{T}\left[\int_0^{T_p}\theta\frac{P-D}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_p}^{T}\theta\frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right)dt\right]$$

Expanding the exponential functions and neglecting second and higher power of θ for small value of θ.

$$=\frac{TD\theta C_p(P-D)}{2P} \qquad (9)$$

(v)Price Discount: In practice, when a supplier is confronted with extreme completion in markets, unanticipated surplus in inventory or change in the production run of a product, he/she may offer a special price discount to motivate buyers to order a special quantity. Price discount is offered as a fraction of production cost for the units in the period $[T_p, T]$

$$\text{Price Discount} = \frac{rC_p}{T}\int_{T_p}^{T} D dt = \frac{DrC_p}{T}(T - T_p) = \frac{D(P-D)rC_p}{P} \qquad (10)$$

(vi) Quality Cost (QC): Quality cost is the cost incurred due to the production of defective items during the first stage of the production cycle

$$QC = \frac{dC_Q T_p}{T} = DxC_Q \qquad (11)$$

Therefore, Total Cost (TC)
= Purchase Cost + Ordering Cost + Holding Cost + Deteriorating Cost+ Price Discount (12)

$$= DC_P + \frac{C_0}{T} + \frac{TC_h D(P-D)}{2P} + \frac{TD\theta C_p(P-D)}{2P} + \frac{D(P-D)rC_p}{P} + DxC_Q \qquad (13)$$

Differentiating the Total Cost w.r.t. T,

$$\frac{\partial}{\partial T}(TC) = \frac{-C_0}{T^2} + \frac{(C_h + \theta C_p)D(P-D)}{2P} = 0 \quad \text{and}$$

$$\frac{\partial^2}{\partial T^2} = \frac{2C_0}{T^3} > 0$$

Therefore, $T = \sqrt{\frac{2PC_0}{D(P-D(C_h+\theta C_P)}}$. (14)

Therefore, Total Cost (TC)
= Purchase Cost + Ordering Cost + Holding Cost + Deteriorating Cost+ Price Discount

$$= DC_P + \frac{DC_0}{Q} + \frac{C_h(P-D)Q}{2P} + \frac{\theta C_p(P-D)Q}{2P} + \frac{D(P-D)rC_p}{P} + DxC_Q$$

Differentiating the Total Cost w.r.t. Q,

$$\frac{\partial}{\partial T}(TC) = \frac{-DC_0}{Q^2} + \frac{(C_h + \theta C_p)(P-D)}{2P} = 0 \quad \text{and}$$

$$\frac{\partial^2}{\partial Q^2} = \frac{2C_0}{Q^3} > 0$$

Therefore, $Q == \sqrt{\frac{2DPC_0}{(P-D)(C_h+\theta C_p)}}$ (15)

3.5 Numerical Example, Let us consider the cost parameters P = 5000 units, D = 4500 units, C_h =10, C_p = 100, C_0 =100, θ = 0.01 to 0.10, C_{PD} =0.05

Optimum solution

Optimum Quantity Q* = 904.53; T_P = 0.1809, T = 0.2010 , Production cost = 450,000 ,
Setup cost = 497.49 , Holding cost = 452.27 , Deteriorating cost = 45.22 ,
Price Discount = 2250 and Total cost = 453244.99

From the table 1, a study of rate of deteriorative items with production time (T_P), and cycle time T and it is concluded that when the rate of deteriorative items increases then the production time, optimum quantity and cycle time decrease, when the rate of deteriorative items increases then the but setup cost, deteriorative cost and Total cost increases.

3.5 Sensitivity Analysis:

The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making chances in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, the current solutions become infeasible, etc.

Observations:

1. With the increase in rate of deteriorating items , optimum quantity (Q*), production time (t_1), cycle time (T) decreases but total cost increases.

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2. With the increase in setup cost per unit (C_0), optimum quantity (Q*), Production time (t_1), cycle time (T) and total cost increases.
3. With the increase in holding cost per unit (C_h), optimum quantity (Q*), production time (t_1) and cycle time (T) decreases but total cost increases.
4. Similarly, other parameters a, production cost, price discount can also be observed from the table-2.

Note : If the production system is considered to be ideal that is no deteriorative are produced, means the value of θ is set to zero. In that case, equations (14) and (15) reduce to the classical economic production quantity model as follows

$$T = \sqrt{\frac{2PC_0}{C_h D(P-D)}}. \text{ Therefore, } Q = \sqrt{\frac{2PDC_o}{C_h(P-D)}}$$

6. Conclusion

This paper presents an EPQ model for deteriorating items in which shortages are allowed and backlogged. The model obtained the optimal values of initial production run time, shortages, production recommencement time and total production quantity that minimizes total relevant costs of production and inventory for any given set of system parameters. In this paper, a dynamic inventory model which is considered with deteriorating production in which each of the production, the demand and the deterioration rates, as well as all cost parameters are assumed to be general functions of time is considered in this paper. The relevant model is built, solved. Necessary and sufficient conditions for a unique and global optimal solution are derived. An illustrative example is provided and numerically verified. The validation of result, this model was coded in Microsoft Visual Basic 6.0.

This research can be extended as follows:

a) Most of the production systems today are multi-stage systems and in a multi-stage system the defective items and scrap can be produced in each stage. Again, the defectives and scrap proportion for a multi-stage system can be different in different stages. Taking these factors into consideration this research can be extended for a multi-stage production process.
b) Traditionally, inspection procedures incurring cost is an important factor to identify the defectives and scrap and remove them for the finished goods inventory. For better production, the placement and effectiveness of inspection procedures are required which is ignored for this research, so inspection cost can be included in developing the future models.
c) The demand of a product may decrease with time owing to the introduction of a new product which is either technically superior or more attractive and cheaper than the old one. On the other hand the demand of new product will increase. Thus, demand rate can be varied with time, so variable demand rate can be used to develop the model.

The proposed model can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time and inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationary stores and others.

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Table 1 : Variation of Rate of Deteriorating Items with inventory and total Cost

θ	Q	T	T_P	Setup Cost	Holding Cost	Deterio-rative Cost	Price Dis-count	Quality Cost	Total Cost
0.01	904.53	0.2010	0.1809	497.49	452.27	45.22	2250	225	453469.99
0.02	866.02	0.1924	0.1732	519.61	433.01	86.60	2250	225	453514.23
0.03	832.05	0.1849	0.1664	540.83	416.02	124.81	2250	225	453556.67
0.04	801.78	0.1782	0.1604	561.25	400.89	160.36	2250	225	453597.50
0.05	774.60	0.1721	0.1549	580.95	387.30	193.65	2250	225	453636.90
0.06	750.00	0.1667	0.1500	600.00	375.00	225.00	2250	225	453675.00
0.07	727.61	0.1617	0.1455	618.47	363.80	254.66	2250	225	453711.93
0.08	707.11	0.1571	0.1414	636.40	353.55	282.84	2250	225	453747.79
0.09	688.25	0.1529	0.1376	653.84	344.12	309.71	2250	225	453782.67
0.10	670.82	0.1491	0.1342	670.82	335.41	335.41	2250	225	453816.64

Note : Production cost constant = 450,000

Table – 2 Effect of Demand and cost parameters on optimal values

Parameters		Optimum values			
		Q	T_P	T	Total Cost
θ	0.01	904.53	0.1809	0.2010	453244.99
	0.02	866.02	0.1732	0.1925	453289.23
	0.03	832.05	0.1664	0.1849	453331.67
	0.04	801.78	0.1604	0.1782	453372.50
	0.05	774.60	0.1549	0.1721	453411.90
C_0	80	809.04	0.1618	0.1798	453139.94
	90	858.12	0.1716	0.1907	453193.93
	100	904.53	0.1809	0.2010	453244.99
	110	948.68	0.1897	0.2108	453293.55
	120	990.87	0.1981	0.2202	453339.95
C_h	8	1000.00	0.2000	0.2222	453150.00
	9	948.68	0.1897	0.2108	453198.68
	10	904.53	0.1809	0.2010	453244.99
	11	866.02	0.1732	0.1925	453289.23
	12	832.05	0.1664	0.1849	453331.67
C_{PD}	0.01	904.53	0.1809	0.2010	451444.99
	0.02	904.53	0.1809	0.2010	451894.99
	0.03	904.53	0.1809	0.2010	452344.99
	0.04	904.53	0.1809	0.2010	452794.99
	0.05	904.53	0.1809	0.2010	453244.99
C_p	80	912.87	0.1826	0.2029	362785.90
	90	908.67	0.1817	0.2019	408015.45
	100	904.53	0.1809	0.2010	453244.99
	110	900.45	0.1801	0.2001	498474.50
	120	896.42	0.1793	0.1992	543703.99